Inverse Problems, Design and Optimization Symposium
Rio de Janeiro, Brazil, 2004

INVERSE PROBLEM OF AIRCRAFT STRUCTURAL PARAMETER ESTIMATION: APPLICATION OF NEURAL NETWORKS

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ABSTRACT

In this paper, a novel method for estimating inertial and stiffness parameters for aircraft structures is presented. The method is based on a combination of the Finite Element Method (FEM) and Artificial Neural Networks (ANNs). ANNs are known for their non-linearity and input/output mapping features and the proposed procedure aims to develop network architecture and training data capable of overcoming many of the shortfalls associated with previous parameter estimation techniques, such as uniqueness of solution and inadequate performance in the presence of uncertainties.

The proposed parameter estimation technique is used to determine inertial and stiffness properties of a linear finite element model comprised of planar Hermitian beam elements. It achieves this with surprising accuracy. The stiffness distribution is estimated from static load/deformation considerations, while the inertial distribution is estimated from the modal characteristics of the model. Finite Element Analysis in MATLAB is used to generate the training data for the networks, which are simulated using its Neural Network Toolbox.

INTRODUCTION

Preface

As the demand for aerospace structures with greater reliability and efficiency increases, so do the levels of complexity and computationally demanding analysis required to engineer them. Classical techniques consistently fail to have adequate robustness and dexterity when adapted to modern engineering problems. There is compelling evidence that Soft Computing techniques like Artificial Neural Networks (ANNs) hold the key to solving traditionally awkward engineering problems by basing them on novel approaches existing in nature. Mathematically speaking, the inverse problem is ill conditioned, hence solution uniqueness is not guaranteed. It is here that traditional techniques begin to falter, and those such as ANNs flourish. Since the task of identifying aircraft structural parameters is an Inverse Problem, the proposed application of ANNs to parameter identification is anticipated to be a powerful and useful means of addressing the many issues that arise when such a taxing task is undertaken.

Literature Review

Currently, there is little research activity involving the application of ANNs to parameter identification techniques for aircraft wing structures, making the research detailed here truly novel. There exists a large research effort into the application of single objective [1] and multi-objective [2] optimisation techniques to the task of wing parameter identification, which for the most part, are "direct" approaches to the problem, however their usefulness has not been discounted. The centre of most of the research regarding parameter identification for aircraft structures is in the area of genetic algorithms [2]. Although being based on Frequency Response Functions (FRFs), which are not pursued in the method proposed here, this work did prove very useful in shedding light as to the major limitations and shortfalls associated with both conventional and unconventional parameter estimation techniques. These were namely the existence and uniqueness of a solution and the "curse of dimensionality".

A technique being researched increasingly in the field of parameter estimation is that of model updating, which seeks to marry the fields of ANN and the Finite Element Method (FEM) [3]. While quite juvenile in its development, it promises to be an exceptionally powerful technique for aircraft wing modelling and parameter estimation. While not directly used in this research task due to its high complexity and advanced nature, it provides a direction for further research activities. There also exists a large body of research regarding static and dynamic modelling of "equivalent aircraft wing structures". By either employing equivalent beam-rod aircraft wing models [4], equivalent plate models [5], or equivalent skin models [6], these techniques aim solely to replace complex physical aircraft wings with simplified and equivalent models that accurately mimic the performance of the actual physical wings. Most of these studies are rather specific and problem dependent in their development, and have the main limitation of being "direct/conventional" approaches to the problem of aircraft wing structural parameter identification, which is an approach avoided here. It is anticipated that while this body of knowledge is not entirely aligned with the proposed research, it still provides useful insight into conventional parameter estimation techniques.

Background into Neural Networks

An ANN is an enormously distributed parallel processing unit, consisting of simpler individual processing units which have inherent tendencies to store and retrieve observed knowledge [7]. ANNs resemble the human brain in that they acquire knowledge and information from their environment which is stored within inter-neuron connections.

ANNs derive their problem-solving prowess from their massively parallel architecture and ability to learn and generalize. They also possess input/output mapping capabilities, adaptivity, robustness and an ability to cope with non-linearity. These traits assist ANNs in solving complex and large-scale (e.g. inverse) problems that are currently unsympathetic to solution.

The Neural Networks utilized here are implemented using the Matlab Neural Network Toolbox. The reader is referred to References [7] and [8] for further information on Neural Networks and their implementation.

PARAMETER ESTIMATION USING ANNs

Model Development

In order to apply ANNs to the estimation of aircraft structural parameters, it is necessary to construct a simplified, but representative model of the desired structural component. As this paper deals solely with the estimation of inertial (ρA) and stiffness (EI) parameters of a cantilevered beam, representative of a real aircraft wing, an appropriate finite element cantilevered beam model was chosen. A schematic of the beam model can be seen in Figure 1.

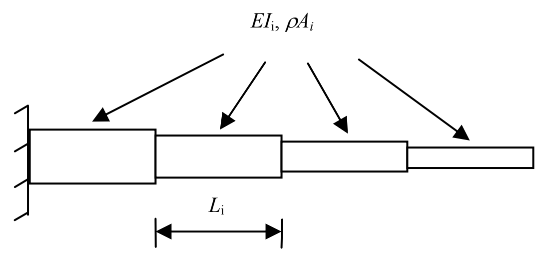

Figure 1. Cantilevered beam model of a real aircraft wing.

Once a suitable model of the wing has been constructed and the properties of the model that are to be identified established, exactly how these properties are to be estimated needs to be ascertained. Hence it is also necessary to obtain some empirical or numerical data regarding the mechanical behaviour of the cantilevered beam, which will be used as a gateway for establishing the desired inertial and stiffness parameters. This is the essence of ANN training. In this paper, two sets of simulated numerical data form the basis of the training data for the ANNs. The first are deformation characteristics of the beam model when subjected to a variety of static loads, whilst the second is the modal characteristics of the beam model from an eigenvalue analysis.

The manner in which this numerical data is used defines the manner in which the estimation problem is formulated. The authors have highlighted three possible approaches to the parameter estimation problem which are depicted in Figures 2, 3 and 4.

Approach 1 involved establishing a single ANN which takes natural frequencies as inputs, and determines (ρA) and (EI) for each beam element in the model. Training data was developed by solving the eigenvalue problem for models with varying mass and stiffness properties. A diagram of this approach is shown in Figure 2.

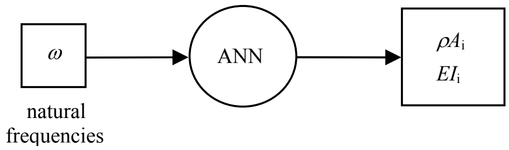

Figure 2. FEM/eigenvalue approach using one ANN.

Approach 2 utilised two ANNs; the first takes natural frequencies and (EI) as inputs, and outputs (ρA), while the second takes natural frequencies and (ρA) as inputs to determine (EI). The training data was developed in the same manner as for the first approach. A diagram of this approach is shown in Figure 3.

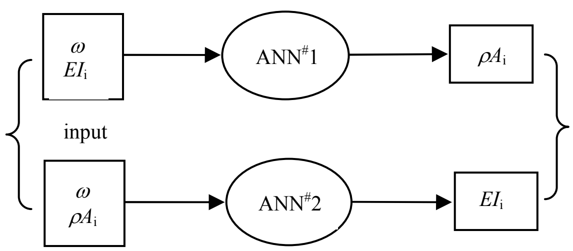

Figure 3. FEM/eigenvalue approach using two ANNs.

In **Approach 3**, (ρA) and (EI) for each element in the beam model are estimated using two different ANNs using two entirely different approaches. The stiffness properties are estimated from static load/deformation considerations, while the inertial properties are estimated from an eigenvalue formulation of the model. Hence two sets of training data are simulated, the first by using Hooke's Law to find (EI) from load/deformation data, and the next by conducting the direct eigenvalue problem for the beam model, calculating the natural frequencies of the beam from (ρA) and (EI). Upon training, one ANN is shown load/deformation data to yield elemental stiffnesses, while the other ANN is shown the previously calculated (EI) values, as well as the natural frequencies of the beam model, to yield (ρA).

Hence the problem reduces to first estimating (EI) for each beam element from load/deformation data of the beam model, and then estimating (ρA) from both the natural frequencies of the beam and the previously estimated (EI) values. This approach is depicted in Figure 4.

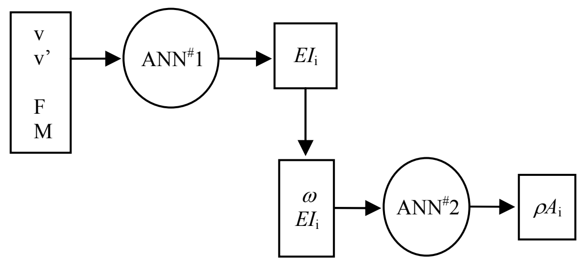

Figure 4. Hooke's Law/eigenvalue approach using two ANNs.

After considerable investigation into which of the three approaches was most feasible for the task considered here, it was found that Approach 3 was the most flexible, more manageable and had fewer inherent deficiencies and limitations. Approach 1 was found to be the most impractical and most difficult to implement. This is mainly due to the fact that, no matter how many elements were used to model the beam, the resulting network was forced to estimate more parameters than it was provided with (the size of the output vector was always greater than the size of the input vector). Such a situation is not at all favourable for ANNs.

Approach 2 attempts to overcome the adverse problems associated with Approach 1 by reducing the size of the output vector and correspondingly increasing the size of the input vector, through the use of two ANNs. However this approach encounters grand problems of its own, which are tied to the nature of eigenvalue problems. The natural frequencies of a mechanical system are dependent on the mass and stiffness distributions of that system, hence the frequencies can be thought of as a function of mass and stiffness. In Approach 2, it is assumed that either the mass or stiffness distribution of the beam is initially known. In general, this will not be the case, and it imposes severe limitations on the applicability and generality of the parameter estimation procedure. Thus it was not considered here.

Hence Approach 3 is the nominated procedure for estimating the inertial and stiffness properties of the beam model. Of all the three approaches, it is the most physically intuitive method, relying heavily on real physical relationships between parameters. This is a very important consideration, since most techniques used to solve inverse problems are very much "black box" approaches, with little regard for the underlying physical relationships relating system parameters. While not entirely "white box" modelling, Approach 3 is an example of "grey box" modelling.

Training Process

Once a suitable model of the beam has been developed, and an approach to solving the task has been decided on, the aspect of simulating training data needs to be addressed. Consisting of input and output pairs, this data is fed into the ANN, which uses it to model the underlying relationship between the input and the output. In this paper, two sets of training data are needed, one for the ANN used to estimate stiffness properties, and another set for the ANN used to estimate the inertial properties. Means of constructing each of these training data sets will be discussed separately below.

Generation of Training Data

The generation of the training data sets for the ANNs is quite similar, and only the set used to estimate the bending stiffness of the beam model will be presented. This data set is produced by solving the Hooke's law problem for many beam models that could be used to represent the real structure. (the other is produced by solving many eigenvalue problems). The procedure for one such beam model is detailed below.

a) Firstly, the bending stiffness (EI) for each element used to represent the beam is generated using a random number generator. This generator produces a number for each (EI) value in the interval [0,1], which is then weighted, so that the values are representative of a realistic tapered cantilever beam (i.e. mass and stiffness properties are reduced in span-wise direction from the supported end of the beam to the free end).

b) Similarly, the magnitude of the slope at each node, for each beam element is generated randomly, and appropriately weighted. Since the beam is to be cantilevered, the condition of zero slope at one end of the beam is enforced.

c) By integrating the slope vector, the deflection at each node can be found. Similarly the condition of zero deflection at one end of the beam is enforced.

d) Next the deflection and slope data are assembled into a global deflection vector, which will be used during the finite element Hooke's Law formulation.

e) The elemental stiffness matrices for the beam model are calculated using the stiffness and geometric data for each element of the beam model. These elemental stiffness matrices are appropriately assembled into a global stiffness matrix, which upon the application of appropriate boundary conditions, will be used during the finite element Hooke's Law formulation.

f) The Hooke's Law problem is solved, using the global stiffness and displacement vectors, to calculate the forces and moments at each node in the beam. Hence the nodal forces and moments for each element in the beam are calculated from the global stiffness and deflection matrices (i.e. $F = Kx$).

g) The force/moment and deflection/slope data is assembled into a vector, which will serve as the input vector for the ANN during training. The output (known during training as the target vector), will be the (EI) values for each element of the beam model. The numbers within each of these vectors are appropriately scaled so that all numbers lie within the interval [0,1], needed for enhanced ANN training.

h) The resulting input/output training data pairs are saved for future network training.

Simulation of Testing Data

In order to assess the performance of the two ANNs at estimating the required structural properties for the beam model, testing data is needed for the two ANNs. This testing data takes exactly the same form as the training data, and is simulated in exactly the same fashion as the two training data sets, except for one subtle difference. The testing data, although resembling the training data, should take on slightly different values, so as to test if the network can generalize sufficiently, and not just memorize the training data. Hence the size of the testing data sets are much smaller than the training data sets, but must fall within the global range that the training data encompasses, otherwise the generalizing process will be compromised.

Network Construction and Simulation

Upon construction of the two sets of training data, the two ANN are constructed and simulated; one ANN used to estimate stiffness properties, and the other ANN used to estimate the inertial properties. Means of constructing each of these ANN are discussed below. Since the construction, training, testing and simulation of each of the two ANN are very similar, each is discussed concurrently.

ANN Used to Estimate Stiffness and Inertial Properties.

These networks are constructed, trained, tested and simulated, producing the required structural properties of the beam model, using the procedure detailed below.

a) Firstly, the relevant training and testing data sets are loaded into the working environment. A global data set of size 2500 is generated for each ANN, which is partitioned into two smaller subsets, one for training and one for testing. Each training data set contains 2475 unique input/output pairs, while each testing set contains 25 unique input/output pairs, sampled at equally spaced intervals from the global data set, which are not seen by their relevant ANN.

Since there are no rigid and well defined procedures to adhere to when determining the size of the training data required for both learning and adequate generalization [7], the optimal size of the training data for each ANN was determined empirically. This meant that a large portion of research involved addressing these issues. It was found that a training data size of 2500 was the minimum possible for efficient network training and adequate generalization (for the most optimum network architecture discussed below).

b) Next the ANN are constructed, by specifying the following-

The **architecture of each ANN**, hence the number of layers and number of neurons in each layer. Both ANN contain two hidden layers, and along with an input and output layer; have a total of four layers. Both ANN contain fifteen neurons in the first hidden layer, ten in the second and four neurons in the output layer.

Two hidden layers were chosen since they represent the lower limit for the solution of inverse problems using ANNs. Since the number of neurons in the output layer must be equal to the size of the output vector, four neurons were used in the output layer. The number of neurons in the first and second hidden layers was empirically determined, again due to the scarcity of routine techniques used for optimal neuron determination. After much consideration between learning efficiency, network performance and training time, for both ANN, the optimum number of neurons in the first and second hidden layers was found to be 15 and 10 respectively.

The **type of activation function** used by each neuron, in each layer for each ANN. The neurons in each of the hidden layers for both ANN use the sigmoid type activation function to accommodate non-linearity, while the output neurons use linear activation functions so that the ANN outputs can take on any real number.

The **training algorithm** to be used for each ANN, along with its performance function, the number of times the training data set is to be shown to the ANN (epochs), along with the stopping criteria. The ANN used to estimate the bending stiffness properties of the beam uses the Levenberg-Marquardt training algorithm, which is an enhanced quasi-Newton numerical optimization training method. The ANN used to estimate the mass properties of the beam uses the Resilient Back Propagation training algorithm, which is an enhanced Steepest Descent training method with modest memory requirements. Both ANN use the Sum Square Error (SSE) performance function to assess network performance, with the stopping criteria being the zero error condition. The total number of epochs for the ANN used to estimate the bending stiffness properties of the beam is one hundred, while the total number of epochs for the ANN used to estimate the mass properties of the beam is five thousand. These epoch numbers ensure adequate network convergence for both ANNs.

The Levenberg-Marquardt training algorithm was chosen for the ANN used to estimate the bending stiffness of the beam since it is accepted to be the fastest method for training moderate-sized feed forward ANNs and has very efficient implementation in the MATLAB environment [8]. However it does suffer from the burden of being very “memory-intensive” if the size of the network is sufficiently large, like the case for the mass estimation ANN. Due to its much larger input vector, the ANN used to estimate the mass distribution of the beam required a different training algorithm. The Resilient Back Propagation training algorithm was found to be the quickest and least “memory-intensive” of the training algorithms available in MATLAB’s Neural Network Toolbox, hence its use in training the ANN used to estimate the mass distribution of the beam. While the Resilient Back Propagation training algorithm is less "memory-intensive" and as fast as the Levenberg-Marquardt training algorithm, it requires more epochs (presentation of training data samples) to learn the underlying relationships within the training data.

c) The ANNs are then trained accordingly, with post training regression analysis carried out to assess the performance of training. The results of network training for the ANN used to estimate the bending stiffness is shown in Figure 5 below.

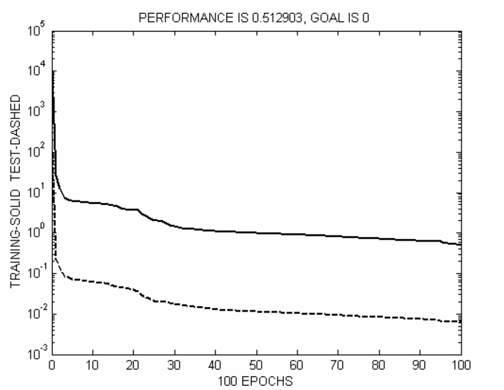

Figure 5. Evolution of network training/testing error for bending stiffness estimation.

With reference to Figure 5, it is evident that the network error goal for the ANN was not met. However after 100 epochs, it can be seen that the network error associated with the ANN has converged to a final value, indicating that further training will not improve network performance. A similar phenomenon occurred for the additional ANN employed for inertial property estimation.

d) The ANN are then presented with the testing data sets, and are asked to produce an estimate of the stiffness and inertial properties for the beam models. Post testing regression analysis is carried out to assess the ANN's ability to generalize. The results of post testing regression analyses for the ANN used for bending stiffness estimation is shown in Figure 6.

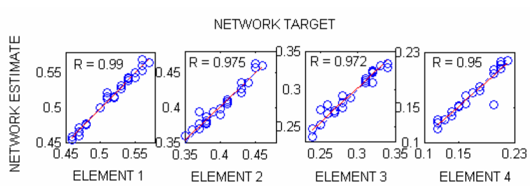

Figure 6. Network post testing regression analysis for bending stiffness estimation.

Figure 6 illustrates the ability of the ANN to estimate the stiffness properties of each element for each beam model used to test the network. This ANN can very accurately estimate the stiffness properties of each element for beam models it has not seen before. This is reinforced upon observation of the correlation co-efficients (R-values) for each element in both the figure, which are all very close to unity, indicating good generalization and accuracy when estimating structural properties of cantilevered beams. Similar results were found for the ANN used to estimate the inertial properties of the beam model.

e) The ANN estimates of the stiffness and inertial properties are compared against the target vectors within the relevant testing data sets, and the relative error is calculated.

f) The ANN and all their associated characteristics are stored for future analysis.

SIMULATED EVALUATIVE EXAMPLE

In order to illustrate the power and robustness of the proposed parameter estimation technique, a simulated example is shown. The model of the aircraft beam is depicted in Figure 1. The beam model is discretised with four Hermitian beam elements, and has one end clamped (no translation or rotation) and the other free to simulate cantilevering. It has a total mass of 287.75 kg and its natural frequencies are listed in Table 1 below. For clarity and simplicity, the mode shape data for the beam, comprising of nodal deflections and rotations at each node is not shown.

In order to estimate the bending stiffness distribution for the beam model, the model is assumed to take up a deflected shape shown in Table 2, when subjected to the loading conditions outlined in Table 2. These correspond to the nodal loads and deformations for the beam model, and form the basis of the input to the first ANN, which is used to estimate the bending stiffness distribution for the beam model.

The loading and deformation data is shown to the ANN used to model the inverse Hooke's Law relationship for the beam, upon which the ANN estimates the bending stiffness of the beam, the results of which are listed in Table 3 below.

Now the stiffness distribution of the beam and its total mass are shown to the second ANN, which is used to estimate the inertial distribution for the beam model, by modeling the inverse relationship existing between eigenvalues of the beam and its structural properties. The results for the ANN used to estimate the inertial distribution of the beam are listed below in Table 4.

Finally, to further test the robustness of the proposed method and validity of results, the first four natural frequencies of the beam model are calculated from the ANN estimates of the structural properties. These are then compared with the actual first four natural frequencies of the beam model to establish the performance of the proposed parameter estimation technique. The results are shown in Table 5.

Table 1. Natural frequencies of the beam model.

Mode	Frequency (Hz)
1	16.41
2	79.62
3	201.87
4	388.89

Table 2. Loading and deformation characteristics.

Location (m)	F (N)	M (Nm)	v (m)	v' (rad)
0	640	-1170	0.0	0.0000
1.25	540	390	2.2	0.0039
2.5	-1070	740	11.6	0.0119
3.75	-1610	440	33.4	0.0232
5	1500	120	67.6	0.0296

Table 3. Bending stiffness estimation results.

Target (Nm^2)	Estimate (Nm^2)	Error (%)
500,000	500,760	-0.15
360,000	360,540	-0.15
260,000	261,360	-0.52
210,000	209,180	0.39

Table 4. Mass estimation results.

Target (kg)	Estimate (kg)	Error (%)
84.2	89.58	-6.39
63.9	65.27	-2.14
49.80	48.25	3.12
32.3	29.99	7.16

Table 5. Accuracy of ANN at estimating beam structural properties.

Target (Hz)	Estimate (Hz)	Error (%)
16.4058	15.5049	5.49
79.6216	73.6173	7.54
201.8719	192.6780	4.55
388.8942	375.2358	3.51

DISCUSSION

It can be seen that upon review of the results presented, the estimated beam parameters represent a beam that closely resembles the real physical beam, with regards to its modal characteristics. Each ANN was able to accurately estimate the properties of the beam model, which in turn led to very accurate estimates of the first four natural frequencies of the beam model.

Advantages

The power of the proposed parameter estimation method lies in its ability to accurately model the generally highly non-linear relationships that are inherent in such structural and dynamic analyses. Hence the proposed parameter estimation technique is not limited to linear static and dynamic systems, greatly enhancing the method's generality and applicability. The input-output mapping capability of ANNs bypasses the need to formulate and work with any highly coupled and non-linear Partial Differential Equations (PDEs) relating the static and dynamic response of the beam to its structural parameters. This greatly facilitates the identification of the required structural parameters for the beam, since highly coupled non-linear PDEs (in general) have no closed form solution and are extremely difficult to solve even numerically. It is indeed the task of each ANN to approximate such equations, which they inherently do in an extremely efficient and accurate manner by learning how the static and dynamic response of the beam relates to the structural parameters of the beam during the training process.

Disadvantages

The main limitation of the proposed parameter estimation method is its heavy reliance on the existence of a significant amount of mechanical data (experimental or numerical) pertaining to the type of structure to be identified. The response of the beam structure to some known static loading regime, as well as the free vibratory characteristics of the structure must be known in advance. This data must be also be converted such that it suitable for use with a simple FE model, which depending on the initial form of the data, may require significant post-processing.

An additional limitation of the proposed parameter estimation method is the use of relatively few beam finite elements in the beam model to represent the aircraft wing. It is well known that by increasing the number of elements used to model the beam, the accuracy of results obtained from the finite element analysis will (to a point) increase, particularly for dynamic analyses. In this case, a small number of elements were used in order to strike a compromise between computational accuracy and efficient implementation of the ANNs. Using more beam elements in the method will require more training and testing data, more hidden layer neurons and longer training times.

Most important however, is the increased computational expense (longer CPU time and increased memory requirement) that accompanies an increase in the number of beam elements used to represent the real beam. For greater than four beam elements, the computational expense of the method becomes overwhelming, while the resulting structural parameter estimates become less accurate, as compared to the four-element beam representation of the physical structure. Hence the use of four Hermitian beam elements in the beam model was identified as providing the most accurate structural parameter estimates for a modest computational cost.

Similarly, since the task at hand was to determine the structural parameters of the beam from a relatively small amount of data regarding its load/deformation and modal characteristics, the simple beam model used in the parameter identification procedure may not be totally representative of the physical structure. Hence the results achieved must be considered light of the numerous assumptions made regarding the loading regime, boundary conditions and geometry of the aircraft wing.

It is therefore apparent that the proposed parameter estimation method can be improved in many ways from the above discussion. Further sophistication of the beam model accompanied with more physical data for the real aircraft wing will lead to more accurate and realistic results. Further post processing of current available real data may lead to an increase in useful data the proposed estimation technique can employ.

CONCLUSIONS

The inertial and bending stiffness distributions of a cantilevered finite element planar beam were accurately estimated using the proposed hybrid FEM-ANN parameter estimation technique. This simple beam model is capable of small deflections and rotations and is treated as a simplified representation of an aircraft wing. The stiffness distribution is estimated from static load/deformation considerations, while the inertial distribution is estimated from the modal characteristics of the beam model. The results from the implementation of this proposed parameter estimation technique show that the estimated parameters produce a beam that has modal characteristics that closely resemble those of the real physical beam. The proposed parameter estimation method showed proficiency at modeling the highly non-linear relationships between input parameters and desired outputs. However it is anticipated that further refinement of the beam model will eventually lead to a model that is more representative of the real structure, for which more accurate results may be obtained.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the DSTO/RMIT Center of Expertise in Aerodynamic Loading (CoE-AL), School of Aerospace, Mechanical & Manufacturing Engineering, RMIT University and the Department of Mechanical and Aerospace Engineering, University of Texas at Arlington.

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